**Calculator Assumed**

**Applications of Differentiation – Rates of Change**

Time: 45 minutes

Total Marks: 45

Your Score: / 45



**Question One: [1, 3, 2 = 6 marks]**

Give an expression for the instantaneous rate of change for each of the following functions, showing all working:

1. 
2. 
3. 

**Question Two: [2, 2, 1, 2, 2, 2 = 11 marks]**

The profit function, in dollars, for producing and selling *x* hundred items is given by :

1. Over what domain does this function adequately model the profit of producing and selling *x* items?
2. What is the profit for producing zero items? Interpret this result.
3. Determine an expression for the instantaneous rate of change for producing and selling *x* items.
4. Calculate and interpret your result.
5. Calculate and compare your result to your answer in part (d).
6. Calculate the average profit for producing and selling the first 200 items.

**Question Three: [3, 3, 1, 5 = 12 marks]**

Consider the function 

1. Sketch this function over the domain on the axes below.
2. Determine the gradient function of and hence calculate .
3. Sketch the tangent to  at  on your graph above.
4. Verify your answer to part (b) by using the limiting chord process and filling in the table below.

|  |  |  |
| --- | --- | --- |
| **Point P** | **Point Q** | **Gradient of segment PQ** |
| (2, 2) | (3, 5) |  |
| (2, 2) | (2.5, \_\_\_\_) |  |
| (2, 2) | (2.1, \_\_\_\_) |  |
| (2, 2) | (\_\_\_\_,\_\_\_\_\_) |  |
| (2, 2) | (\_\_\_\_\_,\_\_\_\_\_) |  |

**Question Four: [2, 2, 2, 2, 2 = 10 marks]**

The function has the following table of values:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 1.1 | 1.2 | 1.5 | 2 | 2.1 | 2.3 | 2.5 |
| *y* | 0 | -3 | -2.97 | -2.88 | -2.25 | 0 | 0.63 | 2.07 | 3.75 |

1. Determine the approximate value of 
2. Determine when  and the value of at this point.
3. Calculate the average rate of change of  between and .
4. Determine the approximate instantaneous rate of change when .
5. Determine the approximate equation of the tangent to when 

**Question Five: [4, 2 = 6 marks]**

The instantaneous rates of change of at ,  and  are given in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | 10 | 10.2 | 11 |
| *T ‘ (x)* | 30 | 31.212 | 36.3 |

1. Determine approximate values of missing in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | 10 | 10.2 | 11 |
| *T (x)* | 100 |  |  |

1. Hence determine the approximate equation of the tangent to  when .

**SOLUTIONS**

**Calculator Assumed**

**Applications of Differentiation – Rates of Change**

Time: 45 minutes

Total Marks: 45

Your Score: / 45



**Question One: [1, 3, 2 = 6 marks]**

Give an expression for the instantaneous rate of change for each of the following functions, showing all working:

1. 

 

1. 

 

1. 



**Question Two: [2, 2, 1, 2, 2, 2 = 11 marks]**

The profit function, in dollars, for producing and selling *x* hundred items is given by :

1. Over what domain does this function adequately model the profit of producing and selling *x* items?

 

1. What is the profit for producing zero items? Interpret this result.

 

1. Determine an expression for the instantaneous rate of change for producing and selling *x* items.



1. Calculate and interpret your result.

 

The profit for producing and selling the next item, i.e. the 201st item.

1. Calculate and compare your result to your answer in part (d).



At the production of 200 items profit was still increasing for every extra item produced and sold, but by 300 items, a loss is being made of each extra item produced and sold.

1. Calculate the average profit for producing and selling the first 200 items.



**Question Three: [3, 3, 1, 5 = 12 marks]**

Consider the function 

1. Sketch this function over the domain on the axes below.





1. Determine the gradient function of and hence calculate .

 

1. Sketch the tangent to  at  on your graph above.
2. Verify your answer to part (b) by using the limiting chord process and filling in the table below.

|  |  |  |
| --- | --- | --- |
| **Point P** | **Point Q** | **Gradient of segment PQ** |
| (2, 2) | (3, 5) | 3 |
| (2, 2) | (2.5, 3.25) | 2.5 |
| (2, 2) | (2.1, 2.21) | 2.1 |
| (2, 2) | (2.05,2.1025) | 2.05 |
| (2, 2) | (2.001,2.002) | 2 |

**Question Four: [2, 2, 2, 2, 2 = 10 marks]**

The function has the following table of values:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *x* | 0 | 1 | 1.1 | 1.2 | 1.5 | 2 | 2.1 | 2.3 | 2.5 |
| *y* | 0 | -3 | -2.97 | -2.88 | -2.25 | 0 | 0.63 | 2.07 | 3.75 |

1. Determine the approximate value of 

 

1. Determine when  and the value of at this point.

 

1. Calculate the average rate of change of  between and .

 

1. Determine the approximate instantaneous rate of change when .

 

1. Determine the approximate equation of the tangent to when 



**Question Five: [4, 2 = 6 marks]**

The instantaneous rates of change of at ,  and  are given in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | 10 | 10.2 | 11 |
| *T ‘ (x)* | 30 | 31.212 | 36.3 |

1. Determine approximate values of missing in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| *x* | 10 | 10.2 | 11 |
| *T (x)* | 100 | 106 | 131 |



 

1. Hence determine the approximate equation of the tangent to  when .

 